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PHYSICS

Three-Dimensional Spinning Waves in the Case of Gas-Free Combustion

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Spinning waves represent a spiral-helix motion of a localized combustion center, which occurs along the lateral surface of a cylindrical sample [1, 2]. After their discovery, these waves stimulated a lot of investigations. Solving unsteady two-dimensional equations of heat conduction and kinetics in the case of combustion of a thin adiabatic shell, authors of [3] have performed for the first time computer modeling of a spinning wave. Then, the two-dimensional approach started to be used in other theoretical studies as well (see, e.g., [4, 5]). However, processes that occurred inside a solid sample in the presence on its lateral surface of a spinning wave remained yet unknown. Attempts to consider three-dimensional models [6, 7] have not yield desired results, because computational algorithms chosen led to rather difficult calculations.

In [8], we have succeeded in a three-dimensional numerical investigation of a spinning wave for a cylinder, having a small radius, and have compared the results obtained with those of the two-dimensional calculation. It was shown that, as in the two-dimensional case, the three-dimensional wave has only one spinning center moving with a constant velocity along a spiral helix. Both models yield close values for the temperature of the center and its dimensions on the surface. The wave period and the center velocity of motion exhibit somewhat stronger differences. In addition, in [8], a new characteristic of a combustion center, namely, its radial dimension was defined. This quantity has a noticeable value, but the radius is smaller than that of the cylinder. While the combustion center moves along a spiral path, its dimensions do not vary. Therefore, its thermal effect on the cylinder axial region is constant in its magnitude (but not in the direction). As a result, combustion of the cylinder central domain occurs in a steady mode that is impossible for the one-dimensional regime of the flame propagation when parameters related to the unstable region have the same values. Furthermore, such a steady-state spinning wave is called classical.

Here, we present results for the numerical analysis of three-dimensional equations that describe gas-free combustion of samples having large radii. New types of spinning waves, which were not described in literature and have undoubtedly three-dimensional nature, have been discovered.

Similarly to [8], we solved the simplest unsteady dimensionless system of equations describing gas-free combustion (according to Frank-Kamenetskiĭ [9]) with the corresponding boundary conditions. For brevity, we do not write out this system of equations here. However, we emphasize that the constitutive parameters of the problem are the following: the dimensionless temperature $\theta = \frac{T - T_*}{\mathrm{Ar}T_*}$, the dimensionless time $\tau = \frac{t}{t_*}$, the dimensionless cylinder radius $R_0 = \frac{r_0}{h_*}$, the Todes criterion $\mathrm{Td} = \mathrm{Ar}\frac{T_*}{T_* - T_0}$, and the Arrhenius criterion $\mathrm{Ar} = \frac{RT_*}{E}$. Here, T is temperature; T_* is the characteristic temperature usually taken to be coincident with the combustion adiabatic temperature; t is time; t_* is the characteristic reaction time; r_0 is the dimensional cylinder radius; h_* is the characteristic width of the reaction zone; R is the gas constant; E is the activation energy; and T_0 is the initial temperature of both the sample and the environment.

We emphasize that, for the chosen temperature scale, the equality $\theta = \theta_{ad} = 0$ corresponds to the adiabatic temperature; the initial temperature is negative ($\theta_0 = -\mathrm{Td}^{-1}$); and the combustion-center temperature is superadiabatic ($\theta > 0$). Details concerning the equations and relations connecting dimensional and dimensionless quantities are presented in [8].

In this paper, similarly to [8], we pay attention mainly to the role of two parameters. They are:

—The depth of the remoteness from the stability limit [10] $\alpha_{st} = 9.1\mathrm{Td} - 2.5\mathrm{Ar}$[1] and

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[1] The quantity α_{st} is inversely proportional to the enthalpy excess in the combustion wave. For the stability limit (critical stability), $\alpha_{st} = \alpha_{st}^{crit} = 1$. In the unstable region, $\alpha_{st} < 1$.

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—The dimensionless radius of the cylinder[2] $R_0 = \frac{r_0}{\mathrm{Td}\Delta z_m}$ [8], where $\Delta z_m = a u_{\mathrm{ZFK}}^{-1}$.

Carrying out the calculations, we paid most attention to the quantity R_0, as the most important one in analysis of not one-dimensional regimes. The role of the heat loss is not considered here. Similarly to [8], the main results following from the analysis of unsteady temperature fields and the conversion depth were calculated with the help of a Pentium-266 personal computer.

Below, we describe characteristic features of newly discovered types of spinning waves.

1. Unsteady one-center spinning waves. The behavior of the spinning center resembles that described in [8] but has an unsteady nature. The dimensions of the center, the velocity of its motion along a spiral helix, and temperature fluctuate. When expanding, the center embraces by its peripheral region a part of the cylinder axis and then narrows again. In contrast to the steady case [8], this causes oscillations of both temperature and an instantaneous combustion rate at the cylinder axis. When the combustion center becomes localized in near-surface layers of the cylinder, the heat flux transferred from it to the cylinder axis is insufficient to cause the transformation of the substance on the axis, because the cylinder has a large radius. However, this flux forms an extensive zone of the warmed-up substance in the cross section orthogonal to the axis. The combustion center approaching this zone initiates its frontal burning. Depending on the constitutive parameters, the temperature maximum can occur at any point of the frontal line, including the cylinder axis as well. After the interior of the sample has burnt, the combustion center again becomes localized near the cylinder surface at which the substance is yet unreacted. That is why, the structure and velocity of the combustion center exhibit temporal periodic variations with periodicity not related to 2π.

2. Two-center spinning waves. There are two spinning centers situated symmetrically. Their motion can be accompanied by either steady-front propagation along the cylinder axis (at small diameters of the samples and insignificant remoteness from the stability threshold) or burning-cylinder central domains in the pulsating regime. In the second case, the behavior of the combustion centers is unsteady. Initially, they expand in the radial direction. As a result, their peripheral regions collide on the cylinder axis, and form a two-headed structure, the temperature increasing in the center of the cylinder. Then, the binary combustion center decomposes into two narrowing parts, and temperature on the cylinder axis drops. The complete separation of a combustion center from the surface does not occur. The periodicity of the unsteady two-headed regime is not connected strictly with 2π. Temperature oscillations in the combustion centers in the cylinder surface areas occur simultaneously (Fig. 1).

We call these regimes conjugate spinning waves.

3. Three-center spinning waves. In contrast to the above-discussed regime, a three-center spinning wave has three combustion centers, which, in turn, leave the surface and move toward the interior of the cylinder. The center, being inside the cylinder, merges with one of the two near-surface centers. As a result, two new combustion centers arise. One of them reaches the surface, while the other moves to the third center. Observing the cylinder surface, we would see disappearance and, then, appearance of the combustion centers. To be more exact, three combustion centers visible on the surface, in turn, flash and, then, lose their brightness. Such regimes can be called flickering spinning waves.

4. Many-center waves. In this regime, four spinning centers that can, in pairs, leave the surface and displace toward the interior of the sample are formed. Moving, they can interact with both each other and the centers staying in the near-surface layer. Possible division of the centers into two parts leads to formation of two-headed structures. The maximum number of the heads observed simultaneously is six. Among the steady-state regimes discovered, this regime is the most complicated, because the spatial inhomogeneity and unsteady temporal behavior are the most clearly expressed in it (Fig. 2).

The features noted above represent only a small part of the information obtained. Discussing it, we present below the data that are, in our opinion, the most important.

First of all, we emphasize that, as the cylinder radius increases, the space-time pattern of spinning-wave propagation becomes more complicated. From the classical steady-state spinning wave, which has one combustion center localized near the cylinder surface [8], we pass to many-center regimes. In these regimes, the velocity of the centers is unsteady, the centers separate from the surface (the phenomenon of flickering) and interact in the sample volume with the subsequent formation of many-headed structures.

We keep the term *spinning* for such regimes, because the velocity of a combustion center preserves the spiral (translational–rotational) component in the cases described. However, the role of this component decreases with increasing R_0, i.e., the spinning pattern of wave propagation becomes more complicated (degenerates). Simultaneously, the role of the radial component of the center velocity becomes more essential, which leads to forming complicated configurations of the unsteady temperature field.

This result is extremely important. It implies that, in the general case, it is impossible to ignore the radial heat transfer in the case of the gas-free combustion of

[2] The quantity u_{ZFK} represents a steady-state wave-propagation velocity given by the Zel'dovich–Frank-Kamenetskiĭ theory; α is the thermometric conductivity; Δz_m is the width of the Mikhel'son pre-ignition zone.

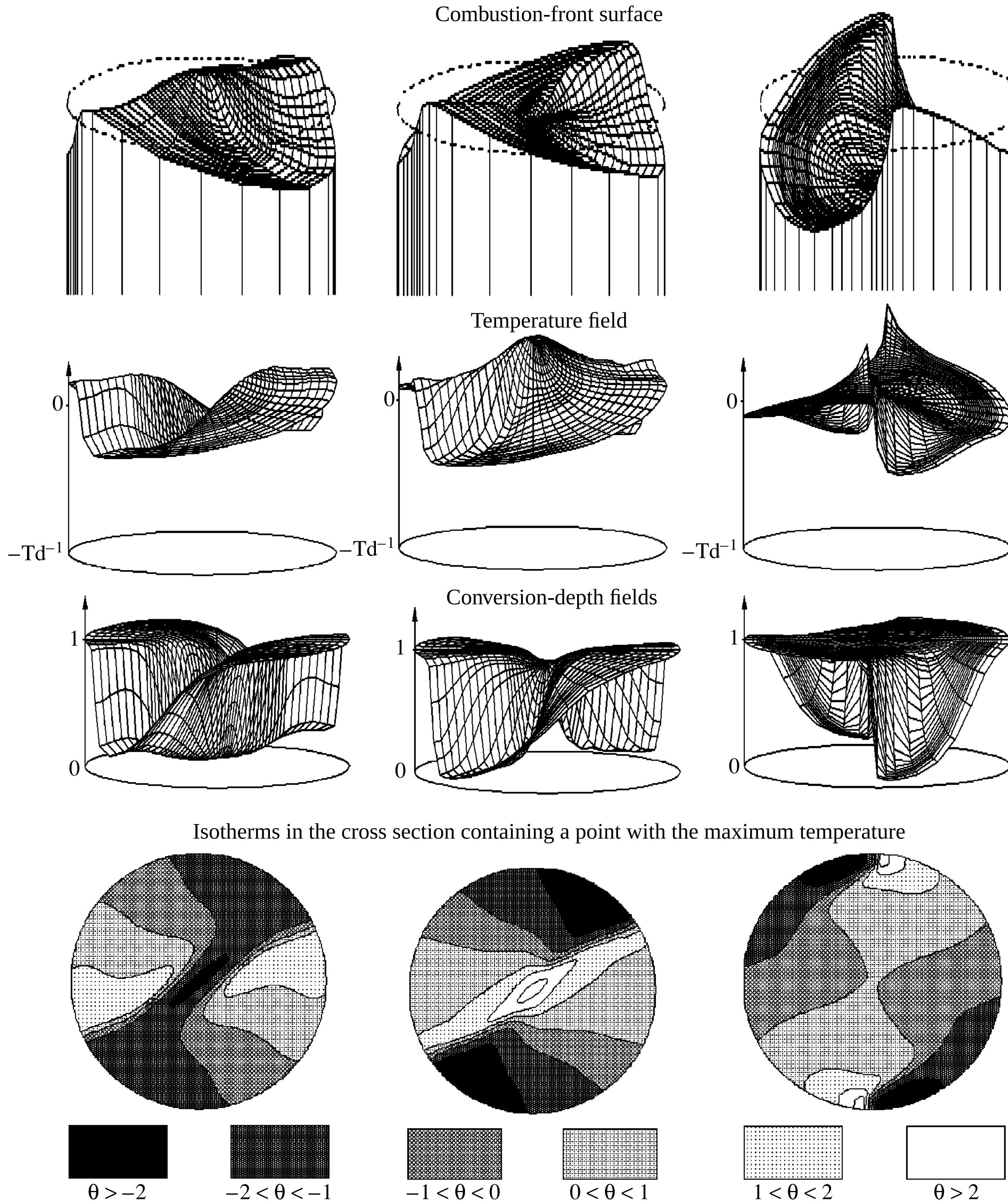

Fig. 1. Conjugate spinning wave at time moments corresponding to the expansion of combustion centers in the radial direction, merging the centers on the axis, and localization of the centers on the surface. Front structure (the dashed line corresponds to the cross section orthogonal to the axis and passing through a point with the maximum temperature), temperature fields and conversion depths, as well as isotherms for the same cross section are shown for each of these moments at $\alpha_{st} = 0.9$ and $R_0 = 67$.

solid samples. Thus, the two-dimensional analysis carried out in [3] and in other studies can be valid only for either samples of small diameters or an artificial model of the adiabatic cylindrical shell.

In all the cases under consideration, even in the case of many-center waves, we deal with the periodic wave propagation. Although, for each set of the values of α_{st} and R_0, the structure and velocity of the wave undergo

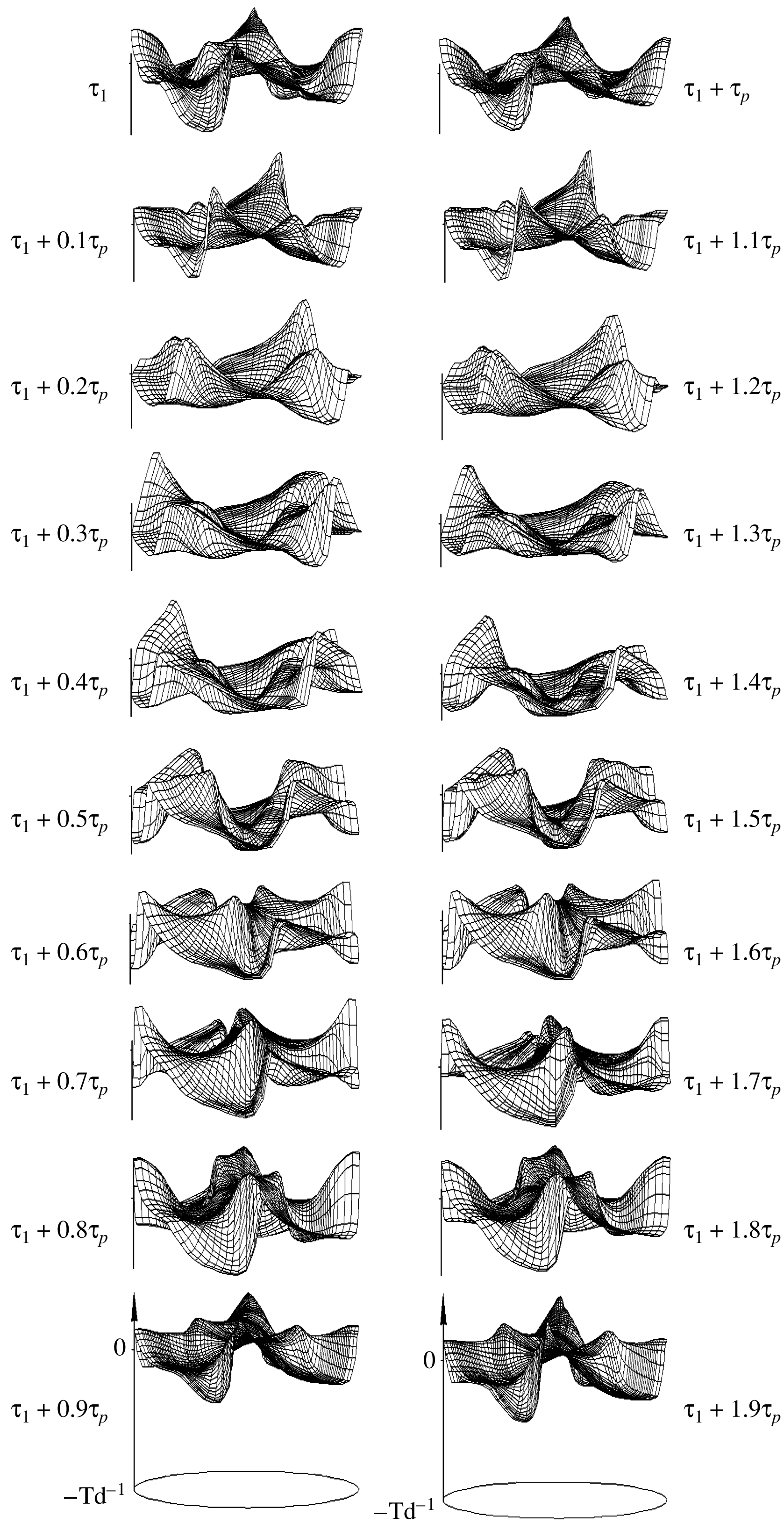

Fig. 2. Dynamics of the temporal variations in the structure of the temperature field for a many-center wave. The patterns corresponds to the cross section orthogonal to the axis and containing a point with the maximum temperature (the point moves downward as the sample combustion occurs). The data are presented with temporal discreteness equal to 0.1 of the wave period τ_p for $\alpha_{st} = 0.9$ and $R_0 = 86$.

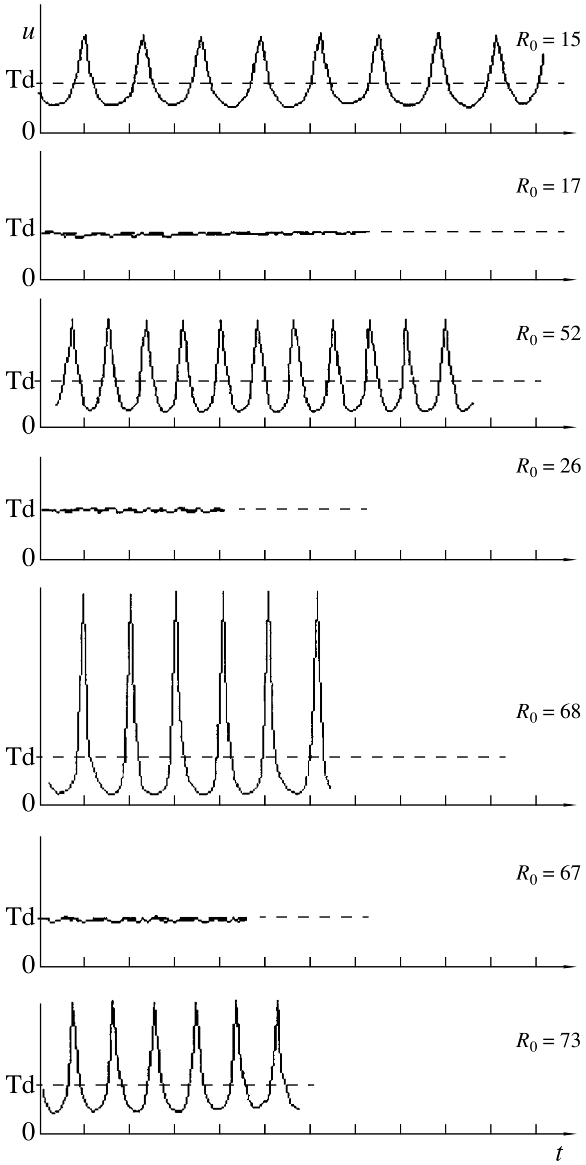

Fig. 3. Temporal dependence of instantaneous translational velocity for the wave propagation along the cylinder axis. The data correspond to different radii R_0 of the cylinder and various regimes occurring at $\alpha_{st} = 0.9$. The following structures are shown from above to below: the plane front ($R_0 \approx 15$); the classical spinning wave ($R_0 \approx 17$); the unsteady one-center wave ($R_0 \approx 52$); the steady conjugate spinning wave ($R_0 \approx 26$); the unsteady conjugate spinning wave ($R_0 \approx 68$), the flickering wave ($R_0 \approx 67$); and the many-center wave ($R_0 \approx 73$). The graduation mark on the time axis is equal to $\Delta\tau = 250$.

diverse variations during the period, the pattern observed being repeated in the next period. Even for the most complicated many-center wave, there exist the propagation periodicity (Fig. 2). Reflecting a tendency to chaotization, the period increases and the front structure becomes more complicated with increasing R_0. The onset of chaos is expected at small values of α_{st} and large R_0. In our opinion, the chaotic regime has the following features: the presence of many combustion centers, variation of their number due to their appearance and disappearance, and an often change in directions of their motion. However, the main feature of the chaotic regime of combustion is the absence of the periodicity in the unsteady structure of the front propagation.

Figure 3 shows instantaneous velocities characterizing wave propagation along the cylinder axis, as a function of time at different α_{st} and R_0. The region of both plane one-dimensional self-sustained oscillations of the front ($\alpha_{st} < \alpha_{st}^{crit} = 1$, $R_0 < R_0^{crit}$) and the spinning waves ($\alpha_{st} < \alpha_{st}^{crit} = 1$, $R_0 > R_0^{crit}$). Since $\alpha_{st} < \alpha_{st}^{crit}$, all the data presented relate to the unstable region. A curious fact was discovered when the effect of the cylinder radial dimension on the combustion regime was investigated. At small R_0, the velocity oscillates periodically near a certain average value $\bar{u}_z \approx$ Td (in accordance with [10]). After the threshold $R_0^{crit} \approx 16$ has been passed, the regime acquires the one-center nature and corresponds to the steady character of the front propagation along the cylinder axis. With further increasing R_0 (and preserving the one-center regime), the oscillations, whose amplitude grows with increasing sample diameters, appear on the cylinder axis again. There is a simple explanation for this, seemingly, unusual effect. When plane self-sustained oscillations occur, the instability is distributed uniformly over the sample cross section, i.e., all of its points are under the same conditions. For a classical-spinning wave, the instability region is localized in cylinder surface layers, and the substance conversion in the interior of the sample is governed by the heat flux transferred from the combustion center moving in the near-surface layers. As the cylinder radius grows, the one-center regime becomes unsteady. At the same time, velocity oscillations arise on the cylinder axis. The larger the cylinder radius, the more considerable the oscillations (Fig. 3). A similar situation arises in the case of conjugate regimes: At small R_0, both the centers on the cylinder surface and the combustion front on its axis move with the constant velocity (Fig. 3). However, with increasing R_0, the unsteady character of the front propagation along both the surface and the axis of the cylinder enhances (Fig. 3).

As is shown in this paper, an increase in R_0 leads also to forming the flickering and other many-center regimes. In these regimes, the character of the front propagation along the sample axis is controlled by the variation of the heat flux from the moving centers.

An important result of our calculations is the nonuniqueness of the steady-state spinning regimes discovered here and occurring at large values of R_0 (Fig. 4). Thus, the existence of two regimes (one-center and two-center waves) is possible at $R_0' < R_0 < R_0''$. More-

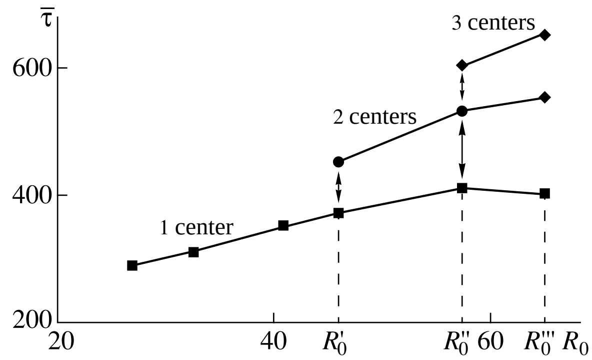

Fig. 4. Non-uniqueness of the steady-state spinning regimes for $\alpha_{st} = 0.86$. Here, $\bar{\tau}$ is the average time of the complete center revolution about the cylinder axis.

over, if $R_0'' < R_0 < R_0'''$, three regimes (one-center, two-center, and three-center waves) can occur. Realization of one or other regime depends on ignition conditions. The non-uniqueness of the spinning waves should be a subject of a more detailed study and needs in substantiation.

The three-dimensional spinning waves occurring in gas-free systems are ill-studied experimentally because of opacity of the samples. A possibility for forming gas-free spinning waves in solid samples has been discovered in [2]. The unsteady one-center spinning wave was realized, seemingly, in [11]. In a number of studies, a set of combustion centers was observed on the sample surface.

Analysis of the calculation results, which was carried out in this paper, allows the conclusion to be drawn that three-dimensional modeling unstable regimes for gas-free combustion is important principally. We consider the information obtained to be only the first step in investigating this complicated problem. Of urgent interest is a deeper study of the instability region for considering the problems of chaotization and chaotic flame propagation, the role of heat loss bounding regions of stable and unstable combustion with respect to various parameters, the thorough investigating nature of the non-uniqueness, and many others.

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